CST-317: Introduction to Earth System Modelling
Exercise II: Exploring SIR Model

The SIR model (from Wikipedia)

In 1927, W. O. Kermack and A. G. McKendrick created a model in which they considered a fixed population with only three compartments: susceptible S(t), infected I(t), and recovered R(t). The compartments used for this model consist of three classes:

- S(t) is used to represent the number of individuals not yet infected with the disease at time t, or those susceptible to the disease.
- I(t) denotes the number of individuals who have been infected with the disease and are capable of spreading the disease to those in the susceptible category.
- R(t) is the compartment used for those individuals who have been infected and then recovered from the disease. Those in this category are not able to be infected again or to transmit the infection to others.

The flow of this model may be considered as follows:

$S \rightarrow I \rightarrow R$

Using a fixed population, N = S(t) + I(t) + R(t), the SIR models uses the following equations:

$$dS = -\alpha * I\ t * S(t)\ dt$$
$$dI = \alpha * I\ t * S\ t - \beta * I(t)\ dt$$
$$dR = \beta * I\ t\ dt$$

Several assumptions were made in the formulation of these equations: First, an individual in the population must be considered as having an equal probability as every other individual of contracting the disease with a rate of α, which is considered the contact or infection rate of the disease. Therefore, an infected individual makes contact and is able to transmit the disease with α*N others per unit time and the fraction of contacts by an infected with a susceptible is S(t)/N. The number of new infections in unit time per infective then is α*N*(S(t)/N), giving the rate of new infections (or those leaving the susceptible category) as α*N*(S(t)/N)*I(t), which is the same as α*S(t)*I(t). This means that the number of people that are infected is set as a fraction of those susceptible.

For the second and third equations, consider the population leaving the susceptible class as equal to the number entering the infected class. The mean daily recovery rate is equal to β, where $1/\beta$ is the number of days it takes to recover from the disease. The parameter β represents the percentage of infected that are leaving this class per day to enter the removed class. Finally, it is assumed that the rate of infection and recovery is much faster than the time scale of births and deaths and therefore, these factors are ignored in this model.

An SIR model with two parameters

In TerraME, we are doing simulations in a discrete time, so we approximate the differential equations of the SIR model by the discrete equations below:

$$S(t+1) = S(t) - \alpha * I(t) * S(t)$$
$$I(t+1) = I(t) + \alpha * I(t) * S(t) - \beta * I(t)$$
$$R(t+1) = R(t) + \beta * I(t)$$

We also calculate the parameters α and β as follows:

- α is the mean infection rate, which is calculated as the product of $\alpha = \gamma * \delta / N$, where γ is the number of contacts of an infected person per day, δ is the strength of the contagion (expressed as a factor from 0.0 to 1.0) and N is the total population.
- β is the mean daily recovery rate, calculated as $\beta = 1/\eta$, where η is the number of days it takes to recover from the disease.

Exercises on the SIR model in TerraME

1) Simple SIR model

Implement a TerraME Model called MySIR. Run the model using the following parameters:

```
scenario1 = MySIR{
     total = 1000,
     infected = 5,
     contactsPerInfectionDay = 1,
     contagionStrength = 0.4,
     infectiousPeriod = 4
}

scenario:run()
```

Explain the graph that you have obtained. What type of disease do you think it is? Calculate a metric called “R0”. Look in Wikipedia and other references, find out what R0 means and explain.

2) A model for measles (“sarampo”)

Measles is a very infective disease that is very common in school-age children. Consider a school with 1000 children where on average each child has close contacts with 20 others and the contagion strength is 20%. Consider that there is an incubation period of 4 days where the child can transmit the infection, but has no external symptoms. Consider that fathers are advised not to send kids to schools if they have the symptoms of measles. Thus, the basic model has the following parameters:

```
scenario2 = MySIR{
     total = 1000,
     susceptible = 999,
     infected = 1,
     contactsPerInfectionDay = 20,
     contagionStrength = 0.2,
     infectiousPeriod = 4
}
```

1. What is the model's prediction for the school? Do all students get the disease? What is the R0 value for this model? Is it compatible with the literature?
2. Compare two ways of reducing the impact of the disease.
   A. A control measure: as soon as 5% of the children of the school have been diagnosed as infected by measles, all children are sent home, thus reducing the contacts between children by a factor of 10 (from 20 to 2). How many children in total would get the disease? Plot the graph, and calculate the total number of infected children.
   B. A prevention measure: almost all of the children have been vaccinated against measles, thus reducing the contagion strength from 20% to 1.5%. How many children would get the disease in this case?

3) A model for influenza ("gripe")

Influenza is an infectious disease caused by the influenza virus. The most common symptoms are chills, fever, runny nose, headache, fatigue and general discomfort. In some cases, as the 1918 Spanish flu, strong strains of the influenza virus can cause significant deaths.

In this example, based on some data from the US, we will use the SIR model for exploring how a flu epidemic could spread in a large city. The basic model has the following parameters:

```
scenario3 = MySIR{
     total = 2500000,
     infected = 100,
     contactsPerInfectionDay = 40
     contagionStrength = 0.015,
     infectiousPeriod = 4
}

scenario3:run()
```

1. What is the model's prediction for the school? Do all students get the disease? What is the R0 value for this model? Is it compatible with the literature?
2. Compare two ways of reducing the impact of the disease.

A. A control measure: as soon as 5% of the people of the city have been diagnosed as infected by influenza, many people stay at home, thus reducing the contacts between infected and susceptible people (from 40 to 20). How many people in total would get the disease? Plot the graph, and calculate the total number of infected people.
B. A prevention measure: children and elderly people have been vaccinated against influenza, thus reducing the contagion strength from 1.5% to 0.85%. How many people would get the disease in this case during one year?

4) Fitting a SIR model to real-life data

This is based on data compiled by the British Communicable Disease Surveillance Centre. The event was a flu epidemic in a boys boarding school in the north of England, lasting from 22nd January to 4th February 1978. There were 763 resident boys, including three initially infected. The data for infected boys in the two-week epidemic are given in the list below.

```
fluData = {3, 7, 25, 72, 222, 282, 256, 233, 189, 123, 70, 25, 11, 4}
```

Build a SIR model for the boys' school epidemic. Find out what are the values for α and β that best approximate the SIR model to actual data. Make some initial assumptions about the infectious period, the number of daily contacts between an infected boy and other boys, and the contagion rate. Adjust these assumptions to obtain a reasonable fit for your SIR model.